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Elastic Lateral-Torsional Buckling of Simply Supported Hot-Rolled Steel I-Beams with Random Imperfections

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Abstract

The paper deals with the problem of lateral beam buckling of simply supported hot-rolled I-beams under major axis bending. The bending stability problem is analysed by consideration the minor axis bending and torsion equations. Both perfect straight beams and beams with initial imperfections are considered. The mathematical solutions are derived, based on a non-linear stability model. The attention is paid to virtual computer experiments of beams with initial random imperfections. Realizations of initial imperfections are simulated, applying the numerical simulation method Latin Hypercube Sampling. Statistical characteristics of the majority of imperfections were found by experimental research. The influence of the beam length on the mean value and standard deviation of load carrying capacity is analysed. The screening based sensitivity analysis method is applied. The principal imperfections affecting lateral beam buckling strength are discussed.

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Keywords: beam, bending, lateral buckling, stability, steel, imperfection, deflection, twist, resistance, failure.

1. Introduction

Structural steel members can be classified as tension or compression members, beams, beam-columns, torsion members or plates [1]. The beams subjected to flexure typically have strength and stiffness in the plane associated with bending about their major principal axis (in the plane in which the loads are applied) much greater than in the plane associated with bending about their minor principal axis [2]. Unless these members are properly braced against lateral deflection and twisting, they are subject to failure by lateral beam buckling prior to the attainment of the full in-plane capacity [3].

The aim of the present paper is a stochastic analysis of bending stability problem of simply supported hot-rolled I-beam with initial random imperfections. Theoretical development and practical applications related to the uncertainty, safety and reliability were obtained both in the field of civil engineering, and in multi-disciplinary approaches including Bayesian methods [4], fuzzy logic models [5], and fuzzy and interval analysis [6], geotechnical engineering and geomechanics [7], geostatistics [8, 9], inspection [10] and quality control [11], optimization under uncertainty [12], probabilistic materials analysis [13], risk analysis [14], vibration [15], reliability-based design [16], reliability-based optimization and control [17], reliability theory [18], statistical design analysis, stochastic computational mechanics, and stochastic finite elements [19], stochastic fracture mechanics [20], system identification [21], system reliability [22], and sensitivity analysis [23, 24] etc.

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2. Buckling of straight beams with equal end moments

The first theoretical studies of the bending stability problem for a beam with thin rectangular cross-section were elaborated, independently of each other, by [25] and [26], namely as early as in 1899. It was S. P. Timošenko [27] for the first time who presented the analysis of lateral beam buckling of horizontally loaded thin-walled steel beams which was of significance for practice. He dealt with the stability of plane bending of the beam with I-cross-section, taking into consideration the bending toughness of flanges at buckling from the plane of primary deformation. Articles from the last period are e.g. [28, 29, 30, 31].

The bending stability problem of hot-rolled I-beam with equal end moment is presented in Fig. 1 and Fig. 2.

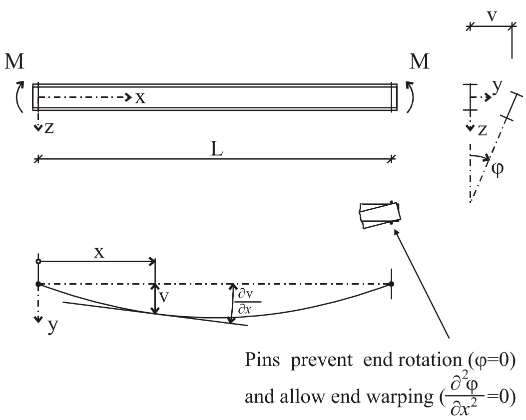

Fig. 1. Lateral beam buckling of simply supported straight I-beam with equal end moment

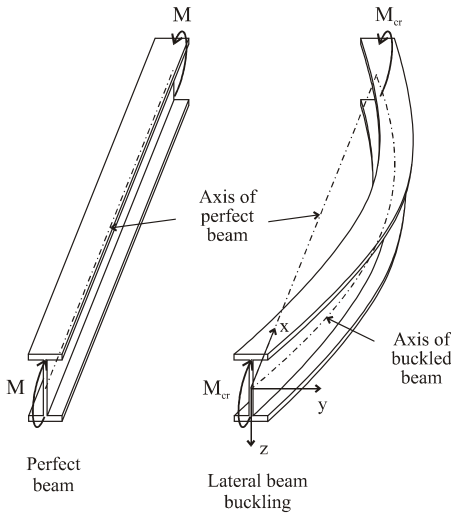

Fig. 2. 3D illustration of lateral beam buckling

The ends of the beam are assumed to be prevented from lateral deflection v and from twisting φ

$$(v)_0 = (v)_L = 0\,,\ (\varphi)_0 = (\varphi)_L = 0\,, \tag{1}$$

but they are free to rotate laterally

$$\left(\frac{\partial^2 v}{\partial x^2}\right)_0 = \left(\frac{\partial^2 v}{\partial x^2}\right)_L = 0\,,\ \left(\frac{\partial^2 \varphi}{\partial x^2}\right)_0 = \left(\frac{\partial^2 \varphi}{\partial x^2}\right)_L = 0\,. \tag{2}$$

The elastic critical moment M_{cr} of the beam shown in Fig. 1 and Fig. 2 can be determined by finding a deflected and twisted position which is one of equilibrium.

Internal minor axis bending moment $M_{cr}\,\varphi$ is caused by applied bending moment M_{cr} and twisted position φ, see Fig. 3. The value of internal bending moment $M_{cr}\,\varphi$ is calculated from the moment equilibrium on deformed beam element, see Fig. 3. The bending moment M_{cr} at any point along the length of the deformed beam is the vector sum of moments $M_{cr}\sin(\varphi)$ and $M_{cr}\cos(\varphi)$. As the value of φ is very low, it can be written approximately $M_{cr}\sin(\varphi)\approx M_{cr}\,\varphi$ and $M_{cr}\cos(\varphi)\approx M_{cr}$.

The torsional moment $M_{cr}\,dv/dx$ is caused by applied bending moment M_{cr} and deflection v, see Fig. 4. The value of internal torsional moment $M_{cr}\,dv/dx$ is calculated from the moment equilibrium on deformed beam element, see Fig. 4. The bending moment M_{cr} at any point along the length of the deformed beam is the vector sum of moments $M_{cr}\sin(\varphi_z)$ and $M_{cr}\cos(\varphi_z)$ where $\mathrm{tg}(\varphi_z)= dv/dx$. Due to the very low value of φ_z it can be written $M_{cr}\sin(\varphi)\approx M_{cr}\,dv/dx$ and $M_{cr}\cos(\varphi)\approx M_{cr}$.

The differential equilibrium equation of bending of the beam is

$$E\,I_z\,\frac{d^2v}{dx^2}=-M_{cr}\,\varphi\,, \tag{3}$$

where E is modulus of elasticity and I_z is second moments of area about axis z.

The differential equilibrium equation of torsion of the beam is

$$E\,I_\omega\,\frac{d^3\varphi}{dx^3}-G\,I_t\,\frac{d\varphi}{dx}=-M_{cr}\,\frac{dv}{dx}\,, \tag{4}$$

where G is shear modulus, I_ω is warping section constant, and I_t is torsion constant. For large torsion, the functions cos(φ) and sin(φ) can be approximated by cos(φ)=1−φ^2/2 and sin(φ)=φ−φ^3/6.

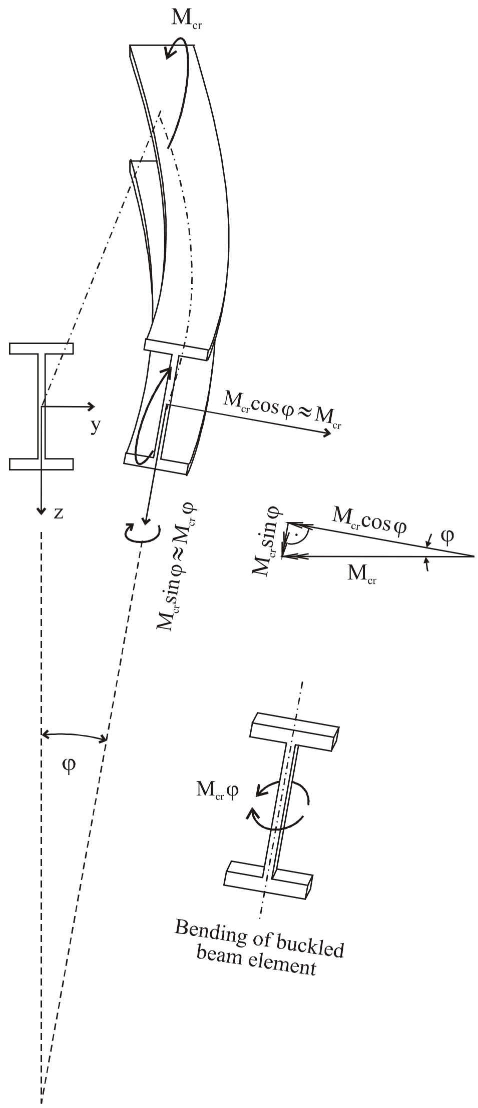

Fig. 3. The bending equilibrium of twisted position of the beam

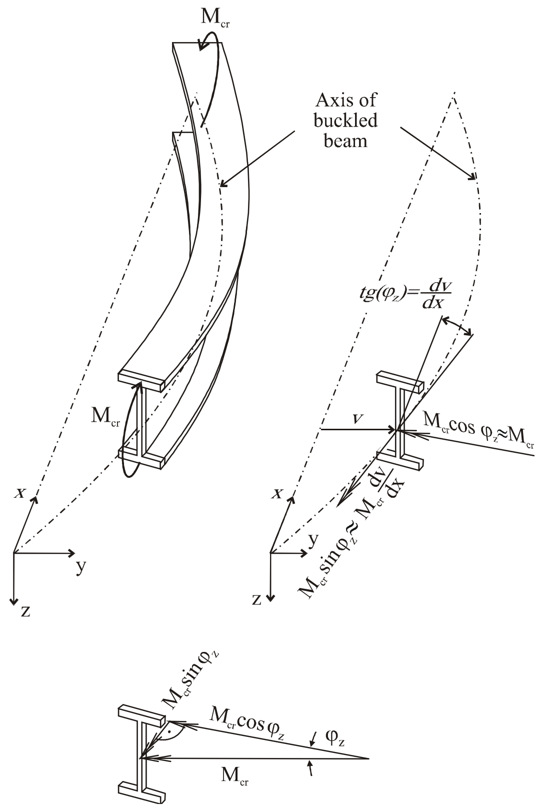

Fig. 4. The bending equilibrium of the beam deflection position

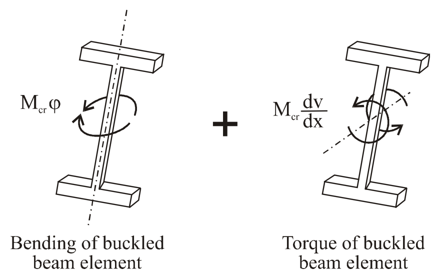

Fig. 5. Components of applied bending moment M_{cr}

Both components of applied bending moment M_{cr} are shown in Fig. 5.

When both (3) and (4) are satisfied at all points along the beam, than the deflected position is

$$v = a_{vc} \sin\left(\frac{\pi\ x}{L}\right) \tag{5}$$

and the twisting position is

$$\varphi = a_{\varphi c} \sin\left(\frac{\pi\ x}{L}\right) \tag{6}$$

where amplitude a_{vc} is the function of amplitude $a_{\varphi c}$

$$a_{vc} = a_{\varphi c}\frac{M_{cr}L^2}{\pi^2 EI_z}\,. \tag{7}$$

By substituting (5) and (6) for (4), we obtain

$$\begin{aligned}&-E\,I_\omega\,a_{vc}\frac{\pi^5 EI_z}{M_{cr}L^5}\cos\left(\frac{\pi\ x}{L}\right)-G\,I_t\,a_{vc}\frac{\pi^3 EI_z}{M_{cr}L^3}\cos\left(\frac{\pi\ x}{L}\right)=\\&-M_{cr}\,a_{vc}\frac{\pi}{L}\cos\left(\frac{\pi\ x}{L}\right)\end{aligned}\,. \tag{8}$$

The elastic critical moment M_{cr} for lateral beam buckling will be obtained from (8) as

$$M_{cr} = \pi\frac{\sqrt{EI_z GI_t}}{L}\sqrt{1+\frac{\pi^2 EI_\omega}{L^2 GI_t}} \tag{9}$$

3. Buckling of imperfect beams with equal end moments

The elastic behaviour of simply supported imperfect beam under equal and opposite end moments M can be analysed with using the minor axis bending and torsion equations.

$$E\,I_z\frac{d^2 v}{dx^2} = -M\left(\varphi+\varphi_0\right), \tag{10}$$

$$E\,I_\omega\frac{d^3\varphi}{dx^3}-G\,I_t\frac{d\varphi}{dx} = -M\left(\frac{dv}{dx}+\frac{dv_0}{dx}\right), \tag{11}$$

where v_0 is initial curvature and φ_0 is initial twist. The ends of the beam are assumed to be prevented from lateral deflection v (1) and from twisting φ (2). The close form solution of (10) and (11) can be solved according to, e.g., [1] and/or [29]. Let us consider the initial imperfection in affine position to buckled shapes (5) and (6) so that the initial curvature v_0 and twist φ_0 are the sine functions

$$v_0 = a_{v0}\sin\left(\frac{\pi\ x}{L}\right) \tag{12}$$

$$\varphi_0 = a_{\varphi 0}\sin\left(\frac{\pi\ x}{L}\right) \tag{13}$$

where amplitudes a_{v0} and $a_{\varphi 0}$ are considered, according to (7), as

$$a_{v0} = a_{\varphi 0}\frac{M_{cr}L^2}{\pi^2 EI_z}\,. \tag{14}$$

The solution of (10) and (11) which satisfied the boundary conditions (1) and (2) is given by

$$v = a_v\sin\left(\frac{\pi\ x}{L}\right) \tag{15}$$

$$\varphi = a_{\varphi} \sin\left(\frac{\pi\ x}{L}\right) \tag{16}$$

where amplitudes a_v and a_{φ} are

$$a_v = a_{v_0} \frac{M}{M_{cr} - M} \tag{17}$$

$$a_{\varphi} = a_{\varphi_0} \frac{M}{M_{cr} - M} \tag{18}$$

The maximum longitudinal stress σ_x in the beam is the sum of the stresses due to the major axis bending, minor axis bending, and warping.

$$\sigma_{x,\max} = \frac{M}{W_y} - \frac{EI_z}{W_z}\frac{\partial^2 v}{\partial x^2} - E\frac{\partial^2 \varphi}{\partial x^2}\omega_{\max} \tag{19}$$

where ω_{max} is maximum of the sectorial co-ordinate of the point used in Vlasov's model for non uniform torsion [32]. For doubly symmetric I-beam is $\omega_{\max} = bh/4$. W_y is section modulus about axis y, and W_z is section modulus about axis z. The failure of imperfect beam is initiated in $x\ 0 = L/2$ when maximum longitudinal stress $\sigma_{x,max}$ causes yield strength f_y. The load carrying capacity (elastic resistance) M_R can be derived from (19) and calculated as

$$\begin{aligned} M_R = &-\frac{\sqrt{4D_1^2 + (D_4 + D_5)^2 + 4D_1(D_4 - 2M_{cr}D_3)}}{4M_{cr}W_z} \\ &+ \frac{2D_1 + D_4 + D_5}{4M_{cr}W_z} \end{aligned} \tag{20}$$

where

$$\begin{aligned} D_1 &= f_y M_{cr} W_y W_z \\ D_2 &= M_{cr} W_z + P_z |a_{v0}| W_y \\ D_3 &= M_{cr} W_z - P_z |a_{v0}| W_y \\ D_4 &= 2P_z^2 I_y |a_{v0}| \\ D_5 &= 2M_{cr} D_2 \\ P_z &= \pi^2 \frac{EI_z}{L^2} \end{aligned} \tag{21}$$

4. Stochastic analysis of load carrying capacity

The load carrying capacity M_R is, in general, a random quantity the variability of which can be studied by applying the numerical simulation methods of type Monte Carlo. 10 000 simulation runs of the Latin Hypercube Sampling method were used. Experimentally obtained material and geometrical characteristics of steel products made by a dominant Czech producer, see [13] were applied to the problem solved. For non-measured quantities, the study was based on data obtained from technical literature; e.g., statistical characteristics of Young's modulus are given in [33]. Input quantities are amplitude a_{v0} of initial curvature, yield strength f_y, cross-sectional height h, cross-sectional width b, web thickness t_1, flange thickness t_2 and Young modulus E. Let us remark that the experimental data are important input information regarding the models of reliability analysis, based on the failure probability calculations [34, 35].

The hot-rolled IPE 240 cross-section was used. All the input factors X_i, given synoptically in Table 1, are statistically independent of one another.

Table 1. Input random quantities

Characteristic	Mean value	Standard deviation
Cross-sectional height h	240 mm	1.0616 mm
Cross-sectional width b	120 mm	1.1842 mm
Web thickness t_1	6.2 mm	0.2421 mm
Flange thickness t_2	9.8 mm	0.4494 mm
Yield strength f_y	297.3 MPa	16.8 mm
Young's modulus E	210 GPa	10 GPa
Amplitude a_{v0}	0	0.76533 L

The principal variable affecting lateral beam buckling strength is the distance between lateral braces. In Fig. 6 to Fig. 23, there is presented the influence of beam length L (unbraced length) on its load carrying capacity.

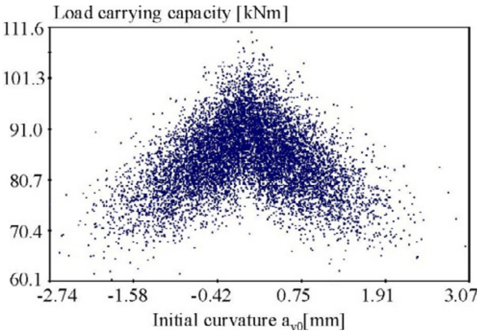

Fig. 6. Load carrying capacity vs initial curvature for L = 1.0 m

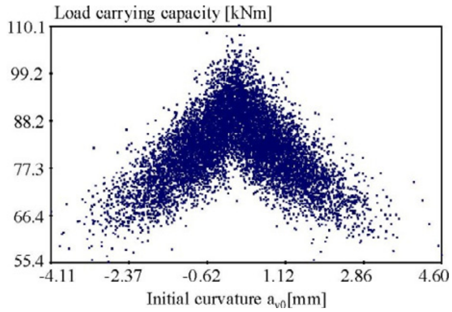

Fig. 7. Load carrying capacity vs initial curvature for L = 1.5 m

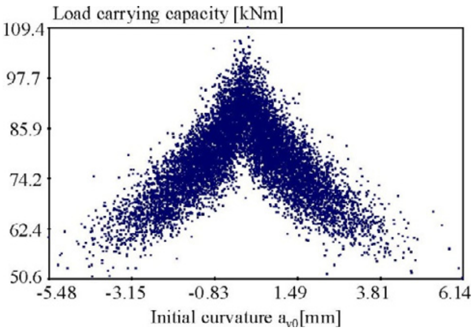

Fig. 8. Load carrying capacity vs initial curvature for L = 2.0 m

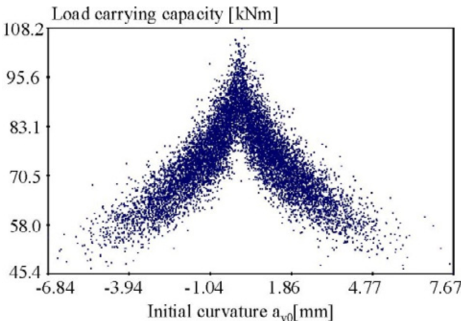

Fig. 9. Load carrying capacity vs initial curvature for L = 2.5 m

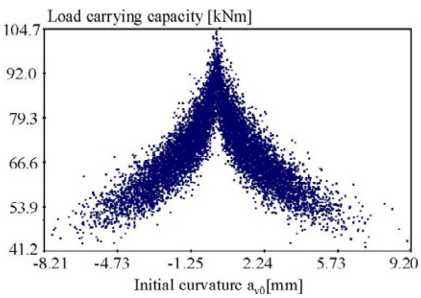

Fig. 10. Load carrying capacity vs initial curvature for L = 3.0 m

The central question of screening in the context of modelling of lateral beam buckling and computer simulation is: Which factors among the input imperfections are really important? The random influence of amplitude of initial curvature a_{v0} on load carrying capacity is evident from Fig. 6 to Fig. 21. The maximum influence is noticeable for the beam lengths $L = 3.0$ m and $L = 3.3$ m. The beam with $L = 3.0$ m has a nondimensional slenderness according to EUROCODE 3 calculated as, $\bar{\lambda} = 0.83$ and the beam with $L = 3{,}5$ m has $\bar{\lambda} = 0.96$. The complete analysis includes Fig. 6 to Fig. 21 plus other 6×16 = 96 figures. Such a screening method is very clear and relatively economical, when compared with the methods of variance based sensitivity analysis. As a drawback, these screening methods do not quantify higher order interactions among input imperfections [36]. Another question arises: How the beam length change influences the load carrying capacity? It is evident from Fig. 22 that the mean value of load carrying capacity decreases with increasing beam length.

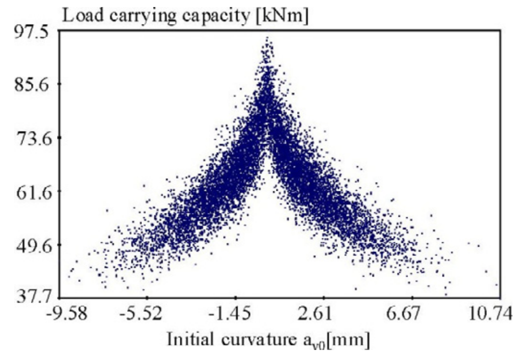

Fig. 11. Load carrying capacity vs initial curvature for $L = 3.5$ m

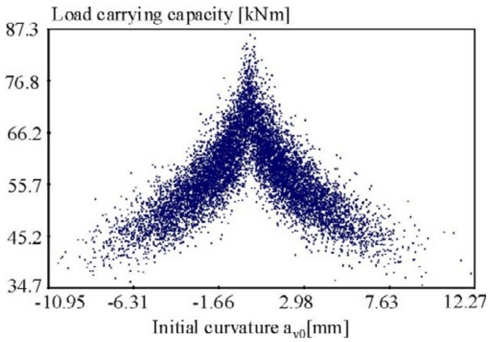

Fig. 12. Load carrying capacity vs initial curvature for $L = 4.0$ m

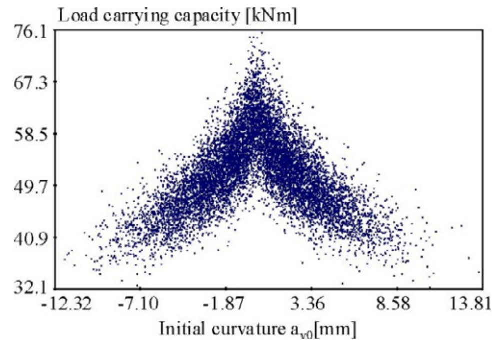

Fig. 13. Load carrying capacity vs initial curvature for $L = 4.5$ m

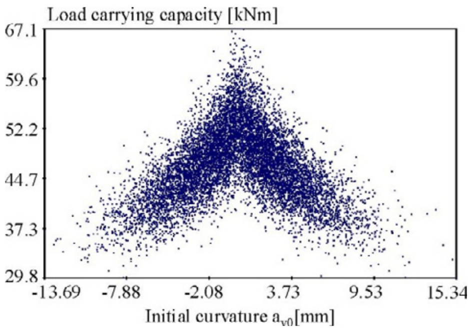

Fig. 14. Load carrying capacity vs initial curvature for $L = 5.0$ m

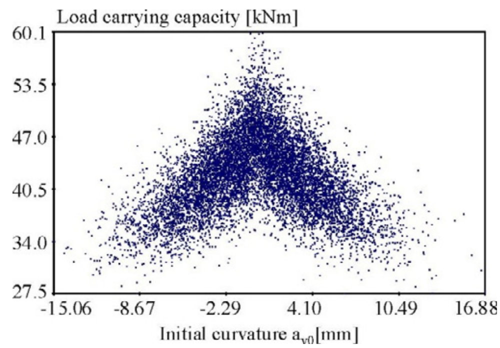

Fig. 15. Load carrying capacity vs initial curvature for $L = 5.5$ m

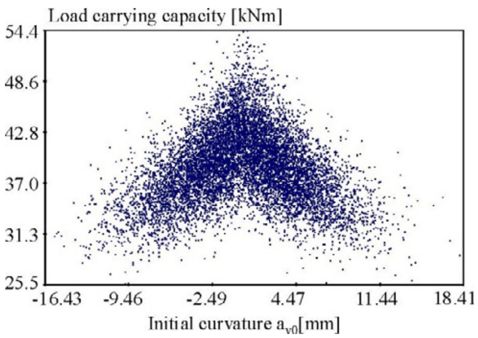

Fig. 16. Load carrying capacity vs initial curvature for $L = 6.0$ m

The analysis of the beam length influence on the standard deviation of load-carrying capacity is depicted in Fig. 23. The maximum of the std. deviation curve was obtained for $L = 2.9$.

The stability design of beams and columns is generally treated in international design codes by using – r all buckling cases – the same curves used_for column buckling (Taras 2010). More general results could be obtained using_the normalized elastic slenderness $\overline{\lambda}_{LT,el}$ for lateral beam buckling [29]. According to EUROCODE 3, non-dimensional $\overline{\lambda}_{LT}$ shall be determined as

$$\overline{\lambda}_{LT} = \sqrt{\frac{W_{pl,y} f_{n,y}}{M_{cr}}} \tag{22}$$

where $W_{pl,y}$ is plastic section modulus and $f_{n,y}$ is characteristic value of yield strength.

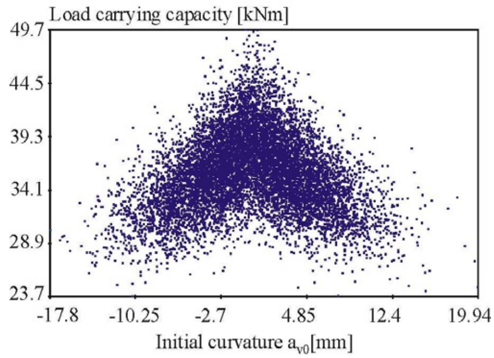

Fig. 17. Load carrying capacity vs initial curvature for L = 6.5 m

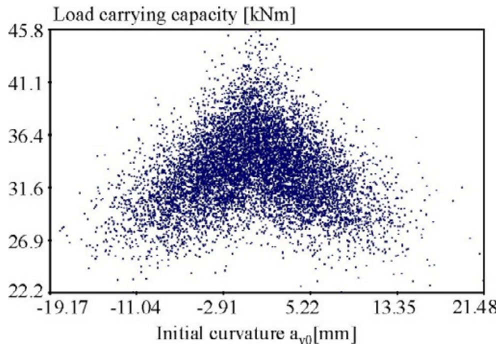

Fig. 18. Load carrying capacity vs initial curvature for L = 7.0 m

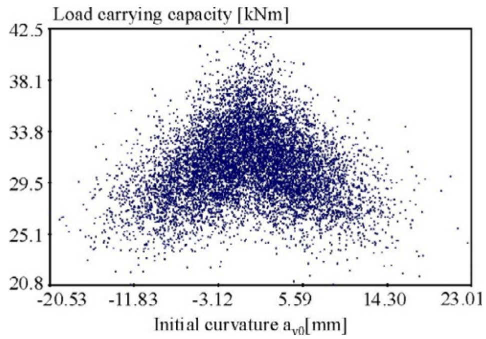

Fig. 19. Load carrying capacity vs initial curvature for L = 7.5 m

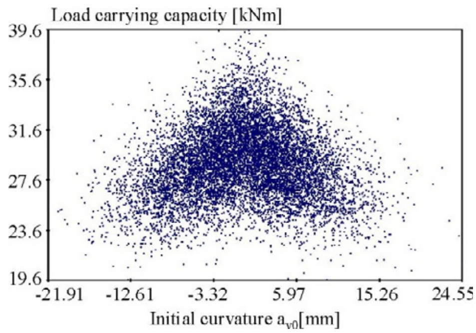

Fig. 20. Load carrying capacity vs initial curvature for L = 8.0 m

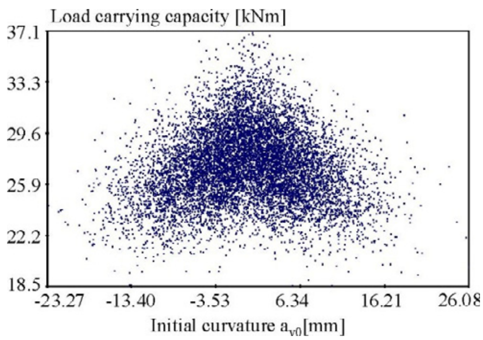

Fig. 21. Load carrying capacity vs initial curvature for L = 8.5 m

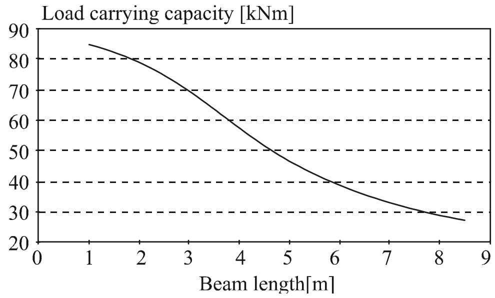

Fig. 22. Mean value of load carrying capacity vs beam length L

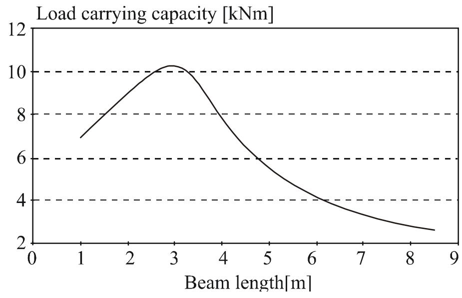

Fig. 23. St. deviation of load carrying capacity vs beam length L

5. Conclusion

Thin-walled elements as beams, columns, trusses or as sheeting rails are extensively used in structural engineering, ranging from residential buildings to industrial construction. Thin-walled structures such as I-section members typically exhibit a detrimental imperfection sensitivity, which drastically reduces their ultimate load bearing capacity compared to their theoretical strength. The problems of bending stability problem of simply supported hot-rolled I-beam with equal end moment were described in the present paper. First, an analysis of idealized perfect straight elastic beams was carried out. The attention was concentrated, above all, on a detailed description of buckled shape geometry which was one of equilibrium. The elastic critical moment M_{cr} for lateral beam buckling was determined. However, a perfectly straight beam may yield before the elastic critical moment M_{cr} is reached because of the combination effect of in-plane bending stresses and initial imperfections. In general, the beam has many imperfections both geometrical and material which can decrease its load carrying capacity. The random influence of initial curvature and initial twist on load carrying capacity was analyzed in the paper. There were determined the beam lengths for which the influence of the imperfections mentioned is maximum.

In further research work, the initial curvature and initial twist will be considered to be statistically independent quantities. The standard deviation of the initial curvature can be more accurately considered under [29]. The standard deviation can be derived according to the limit of the initial curvature, see Fig. 24.

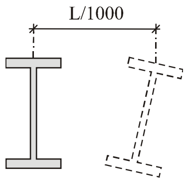

Fig. 24. Possible maximum value of the initial deformation

The initial twist can be approximately ±1°. Is the influence of initial twist very low or could it be totally neglected? This hypothesis should be verified by a computer model based on shell finite elements and nonlinear computation methods.

Acknowledgements

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